A Subspace Clustering Chart Using a Reference Model for Featureless Bearing Performance Degradation Assessment

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Abstract: The health index (HI) of machine condition must be sensitive and robust in complex working conditions. A systematic HI will assess machine performance automatically, reliably, and in a timely manner without intervention. This paper proposes a subspace clustering HI in a model using reference data on component health. Unlike the conventional HIs empirically learned from raw feature sets, a subspace clustering HI aims to automatically describe the migration and variation of the condition clustering distribution in a series of two-class subspace models derived from the raw data. First, in a featureless process, a covariance-driven Hankel matrix is directly constructed from the raw time-domain signal, and principal component analysis is used to separate the feature subspace and noise null-space. Second, in the index construction process, the reference health subspace data (from healthy data) and the monitored subspace data (from monitored data) are combined to construct a referenced model. Thus, a new spatial clustering HI with kernel operation is implemented to assess the current bearing performance and reveal discriminative features. The effectiveness of the proposed subspace clustering HI for the detection of abnormal condition is evaluated experimentally on bearing test-beds, using a mobile mapping mode. A novel subspace clustering chart, CUSUM-based spatial clustering HI, is developed to depict the real bearing performance degradation. Compared to the regular HI (e.g., root mean square), the proposed approach provides a more accurate and reliable degradation assessment profile with an early fault occurrence alarm. The experimental results show the potential of the proposed spatial clustering analysis to assess bearing degradation.

Key words: Bearing health monitoring; spatial clustering analysis; subspace clustering chart; featureless process; performance degradation assessment

Introduction: Bearings are widely used in rotating machinery but are easily damaged, creating a need to reliably assess their health in a timely manner [1-2], so that failure and loss can be prevented. Ideally, health degradation indicators will monitor and accurately assess a bearing's health condition throughout its lifecycle, giving early warning of slight degradation. But constructing a reliable and sensitive health indicator (HI) remains an elusive challenge. The dominant analyses are time-domain, frequency-domain and time-frequency analysis, with feature selection (FS) [3-4] or feature extraction (FE) [5-6] techniques often used as pre-processing methods for bearing fault diagnosis and degradation assessment.

It should be noted that, first, in conventional health monitoring schemes, these features are constructed by means of artificial experience and intervention. Second, the HIs are always directly constructed or mapped from a healthy dataset in the training process. The new obtained features are put into models for bearing fault diagnosis [7-8], performance assessment [9-11] and residual useful life (RUL) prediction [12-13]. For example, in the application of bearing health monitoring, Qiu et al. [9] trained self-organizing map (SOM) from the original feature matrix of the normal data and used the minimum quantization error (MQE) to detect the degradation. Yu [10] obtained the feature mapping from the healthy data by performing locality preserving projection (LPP) and put the mapped features of the monitored data into a Gaussian mixture model (GMM) to determine bearing performance degradation. Pan et al. [11] used the extracted features from normal data to train a support vector data description (SVDD) model and health index (HI) based on general distance. The above work mainly focuses on artificial features built from raw data and relies on mathematical models for assessment.

The following two characteristics are not well considered in the assessment process. First, according to the physical mechanism of the bearing failure, there are differences between abnormal and normal data. Second, the principle of statistical quality control indicates that the feature distribution of the real-time monitoring data will transfer from heathy to degraded status with operating time. Based on this understanding, our previous work [6,14] has shown that a significant difference between the features of healthy and unhealthy signals can improve the clustering and distinguishability of multiclass features, including identification of different degradation severities with similar feature distribution. Arguably, then, a reference model using healthy data with feature clustering distribution can be imported into performance degradation assessment (PDA).

However, artificial statistical characteristics need to be prepared in advance, and conventional vector features are distributed by noise and external interference. To detect structure damage, subspace identification (SI) [15,16] is used to mine the principal subspace, so that the desired information can be well represented and identified. Due to its simplicity and utility, this study employs SI to automatically mine the latent components of health condition. It uses a series of subspace bases to support a two-dimensional space and operating time, so that the monitoring process is a three-dimensional feature interpretation process.

The paper uses the subspace clustering distribution transition healthy data in a reference model to assess the condition of the bearing in a process called spatial clustering analysis. It proposes a subspace clustering chart using the reference model instead of the complex mathematical monitoring models with artificial features. First, a series of subspace bases are automatically reached through manifold learning from a covariance-driven Hankel matrix. Then, a reference model is built using the same reference data (healthy data) and the monitored samples at each moment. Finally, a new subspace clustering HI is implemented in Grassmann manifold space to assess the current machine performance. In this manner, a subspace clustering HI can be obtained for a featureless health condition assessment; the latent components of each health condition can be described in an automatic mode, with variations in information highlighted by the reference model. In short, spatial clustering analysis and an improved subspace clustering chart using a reference model with healthy data are proposed for featureless bearing health performance degradation identification and assessment.

The rest of this paper is organized as follows. Section II presents the framework of the subspace clustering chart using a reference model; a covariance-driven Hankel matrix is automatically constructed for spatial clustering analysis. Section III introduces the experimental setups; abnormal information is detected using a mobile mapping mode and bearing performance degradation is assessed over the full lifecycle to verify the effectiveness of the proposed model. Section IV provides conclusions.

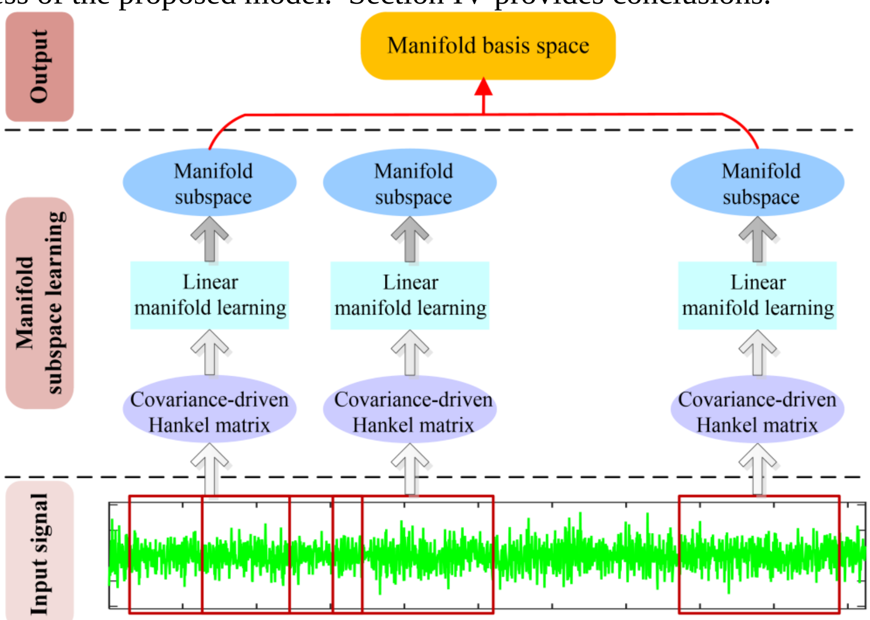

Figure 1 Flow chart of the manifold basis space construction.

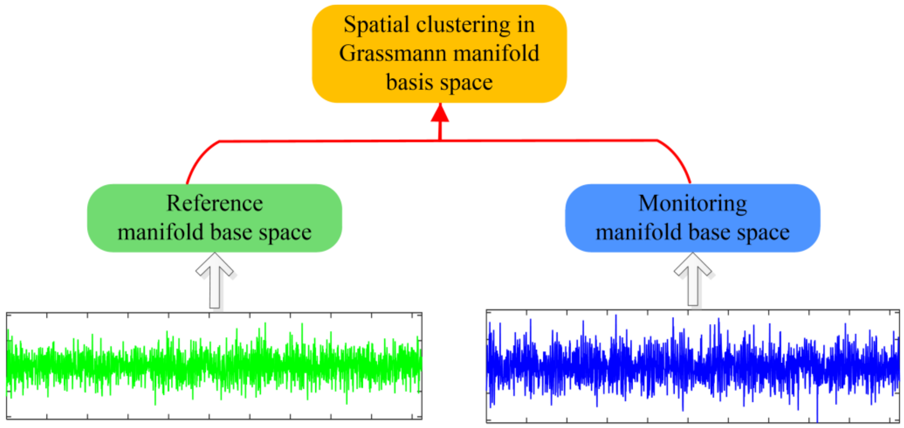

Figure 2 Flow chart of the featureless cluster analysis based on Grassmann manifold basis space.

Subspace clustering chart using reference model: The accuracy of health identification depends on the sensitivity of the extracted features and their ability to show variations from the healthy state. However, because of the complex conditions and because early degradation has a weak response, it is difficult to achieve a timely and reliable performance degradation assessment using artificial features in a conventional monitoring process. Therefore, this paper proposes a subspace clustering chart using a reference model with healthy data to monitor and assess bearing health. The process involves two steps: manifold base space construction (MBSC) and featureless clustering analysis (FCA) based on the Grassmann manifold (GM) reference model. Detailed flowcharts are shown in Fig. 1 and Fig. 2.

In the process of MBSC, as illustrated in Fig. 1, a new Hankel matrix is built from the raw time-domain signal. Then, the manifold subspace is adaptively learned from these matrixes through manifold learning. Lastly, these are employed to support the new manifold basis space, representing the part's present condition. In the process of performing the FCA of the Grassmann manifold reference model, as displayed in Fig. 2, a new reference model is built combining the healthy manifold basis space and the monitored manifold basis space. A discriminant index is later calculated to assess the spatial clustering of these two manifold bases in GM's space. A subspace clustering chart is thus obtained for online health monitoring. Unlike the classical health monitoring scheme, with the healthy data, the feature clustering distribution of the reference model can expose differences once a fault happens.

The following sections provide more detailed information on the proposed subspace clustering system, including subspace matrix construction, manifold learning, spatial clustering analysis, and CUSUM-based subspace clustering chart.

Covariance-driven Hankel matrix construction: In order to achieve an adaptive state representation from the raw signal, this paper employs subspace identification (SI) to separate and mine the desired structural subspace to construct the principal-subspace or null-subspace supported by the signal space. SI utilizes the incoherence between the feature information and noise components in the space domain. The proposed covariance-driven Hankel matrix is operated in this manner. For a given multi-channel signal, X={x_1, x_2,…, x_N} ϵ $R^{T\times N}$ (T and N are respectively the channel and length of the measured signal), a covariance-driven Hankel matrix can be obtained as:

$$\mathbf{H}\big|_{p\times q}=\begin{bmatrix} C_1 & C_2 & \dots\dots & C_q \\ C_2 & C_3 & \dots\dots & C_{q+1} \\ \dots & \dots & \dots\dots & \dots \\ C_p & C_{p+1} & \dots\dots & C_{p+q-1} \end{bmatrix} \tag{1}$$

where p and q are the size of the matrix, p is equal to q, and C_i is the output covariance, defined as:

$$\begin{aligned} C_i &\approx \frac{1}{N-i+1}\sum_{n=1}^{N-i+1} x_n x_{n+i-1}{}^T \\ &= \frac{1}{N-i+1}[x_1 \quad x_2 \ \dots \ x_i]\big|_{S\times i}\cdot[x_i \quad x_{i+1} \ \dots \ x_N]^T\big|_{i\times S} \end{aligned} \tag{2}$$

Based on the matrix **H**, the corresponding principal subspace of the monitored signal can later be mined to assess the equipment condition.

Manifold learning: The latent manifold of the raw feature set can be mined in linear and nonlinear learning processes. Considering the parameters and efficiency with orthogonal bases, principal component analysis (PCA) is applied to obtain the latent condition information for the principal subspace remaining. Thus, a new manifold basis space can be built using d principal basis written as:

$$S = span[s_1, s_2, ..., s_d] \in R^{p\times d}, d \le p \tag{3}$$

Note that subspace S represents condition information in a high-domain information space; this will be further analyzed in Grassmann manifold (GM) space [17].

Spatial clustering analysis: Along with the working time, data on health status can be collected. To explore the health status and determine differences between the healthy and monitored status, a reference model with two classes is built at each moment. In other words, two kinds of basis spaces are combined after the corresponding manifold basis spaces are obtained. This means that a two-class basis space matrix at monitored moment t (t=1,2,…,T) can be created by synthesizing the set $\mathbf{X}_{\mathrm{health}}$={$S_1$, S_2, …, S_{n0}}, with n_0 number of samples, together with the corresponding basis space set $\mathbf{Y}^t_{\mathrm{monitor}}$={$S^t_1$, S^t_2,…, S^t_{nt}}, with n_t samples. Thus, as shown in Fig. 2, a reference model $\mathbf{Z}_t$={$\mathbf{X}_{\mathrm{health}}$, $\mathbf{Y}^t_{\mathrm{monitor}}$} using healthy data is constructed for the subsequent spatial clustering analysis.

Instead of using the traditional one-dimensional feature vector from a time-domain signal, this paper uses a two-dimensional basis matrix based on MBSC with Riemannian metric. A new spanning basis matrix can be regarded as a new point and a series of bases can support a subspace called Grassmann manifold (GM). The aforementioned manifold basis space can be regarded as a new point in this subspace group, $O(n)$ ϵ G^n, and the relationship between these subspaces can be represented as:

$$d_G(\mathbf{P},\mathbf{Q}) = \|\boldsymbol{\theta}\|_2 \tag{4}$$

where is θ=[θ_1, θ_2,…, θ_{m}] and usually calculated as:

$$\begin{gathered}\cos(\theta_k) = \max_{\mathbf{p}\in\mathbf{P},\mathbf{q}\in\mathbf{Q}} \mathbf{p}^T\mathbf{q} = p^T q \\ s.t.\ \ \|\mathbf{P}\| = \|\mathbf{Q}\| = \mathbf{I}_m\end{gathered} \tag{5}$$

From Eqs. (3)~(5), it can easily be seen that an operation is achieved by utilizing an inner product. Therefore, as in manifold learning, a kernel operation is used to retain the latent information, such as KPCA, LPP etc. In this study, we can suppose that the desired information is embedded in the GM of the reproducing kernel Hilbert space (RKHS). Three kernel operations are usually applied to the GM spaces: Binet-Cauchy kernel [18], projection kernel [19], and correction pseudo kernel [20], respectively defined as:

$$d_{bc}\left(\mathbf{P},\mathbf{Q}\right) = \det\left\|\mathbf{P}^T\mathbf{Q}\mathbf{Q}^T\mathbf{P}\right\| \tag{6}$$

$$d_p\left(\mathbf{P},\mathbf{Q}\right) = tr\left(\mathbf{P}^T\mathbf{Q}\mathbf{Q}^T\mathbf{P}\right) \tag{7}$$

$$d_{cc}\left(\mathbf{P},\mathbf{Q}\right) = \max_{p\in\mathbf{P},q\in\mathbf{Q}}\left(p^T q\right) \tag{8}$$

For a given sequence of time-domain signal $x \in \mathrm{R}^{1\times N}$, the manifold basis space S=span{s_i, i=1,2,…,d} $\in \mathrm{R}^{p\times m}$ can be obtained to construct the reference model. To evaluate the distribution of the reference model $\mathbf{Z}_t$, a spatial clustering factor is further proposed in the GM space with a kernel operation as mentioned above, found as:

$$G = G_b / G_w \tag{9}$$

$$G_b = \sum_{j=1}^{c}\sum_{k\neq l\in C_j} d(\mathbf{S}_k, \mathbf{S}_l) \tag{10}$$

$$G_w = \sum_{j=1}^{c}\sum_{l\in C_k, k\neq j} d(\mathbf{S}_j, \mathbf{S}_k) \tag{11}$$

where the GM-based between-class scatter G_b indicates the scattered level among different classes, while the GM-based within-scatter S_w describes the concentrated level in the same class. The GM-based discriminant factor G is a comprehensive indicator that combines the property of between-class scatter and within-class scatter. Specifically, according to Eqs. (6)~(8), three different GM-based discriminant factors can be acquired, respectively called G_B, G_P and G_C. This GM-based discriminant factor G can be used to trace and assess the distribution and transition of the manifold spaces as these changes due to performance degradation. It is foreseeable that this HI will be suitable for the detection of abnormal status. Thus, a new subspace clustering chart can be created for health monitoring.

Experimental descriptions: To verify the effectiveness of the proposed spatial clustering analysis in monitoring equipment status, two cases are tested here. One is the detection of abnormal status using a mobile measurement system to acquire the acoustic signal of a bearing; the other is the assessment of bearing performance degradation for the full lifecycle of the bearing using a test provided by PCE, IMS of University of Cincinnati. The details of two cases are given below.

Case I: First, to validate the ability of the proposed method to detect abnormal status, a mobile measurement system is built to acquire the acoustic signal of a ball bearing (Type 620 6-2RZ HRB) with an outer-race defect, as shown in Fig. 3(a). Fig. 3(b) illustrates a sound mobile measurement system, where sound signals from mobile multipaths are collected by the front-end collector in a hand-held inspection. This information is transmitted to the host computer to detect the mobile health condition. In this case study, two android mobile phones (ZTE 889D and MI 1) with specially developed acquisition software are employed to movably acquire the acoustic signal in a random way. The sampling frequency is 44.1 kHz with a sampling time of 10s.

Two different experiments (Data A and Data B) are tested: one is for mobile detection with different measuring locations, as shown in Fig. 3 and Table 1, and the other is for mobile detection with different measuring locations and tools, as in Table 2. It should be noted that two status conditions, healthy and outer-race detect, are tested. In the test for Data A, two traditional HIs, namely RMS and kurtosis, are calculated for 70 samples; their corresponding scatter plots are displayed in Fig. 4. There seems to be no valid knowledge from these two status conditions; they distribute four different scatter classes, leading to a confusing detection. There is an even worse phenomenon in Fig. 4(b). Because there were unavoidable impacts in the bearing test, the kurtosis failed to detect impact. Based on the proposed spatial clustering analysis, three HIs, G_B, G_P and G_C as introduced in Eqs. (6)~(11), are respectively obtained on the GM-based reference model. The scatter results are displayed in Fig. 5. It can be easily seen that these three HIs yield a much better clustering effect; the same types of status condition can be gathered together and normal and abnormal conditions can be clearly distinguished.

In the test for Data B with RMS and kurtosis, using the different measuring locations and tools, confusing detection results are obtained, as shown in Fig. 6. Next, using the proposed method, the GM-based subspace clustering HI is calculated, as shown in Fig. 7. According to the halving line between two status conditions, G_P and G_C, the method outperforms conventional RMS and kurtosis. By introducing the reference model using healthy data, the proposed spatial clustering HI can detect the difference between normal and abnormal conditions while ignoring the effect of the measuring distance, measuring tools, and some inference from the working conditions. Therefore, the proposed spatial clustering using the reference model can achieve mobile detection in an itinerant way and shows good potential for use in real production quality control (PQC).

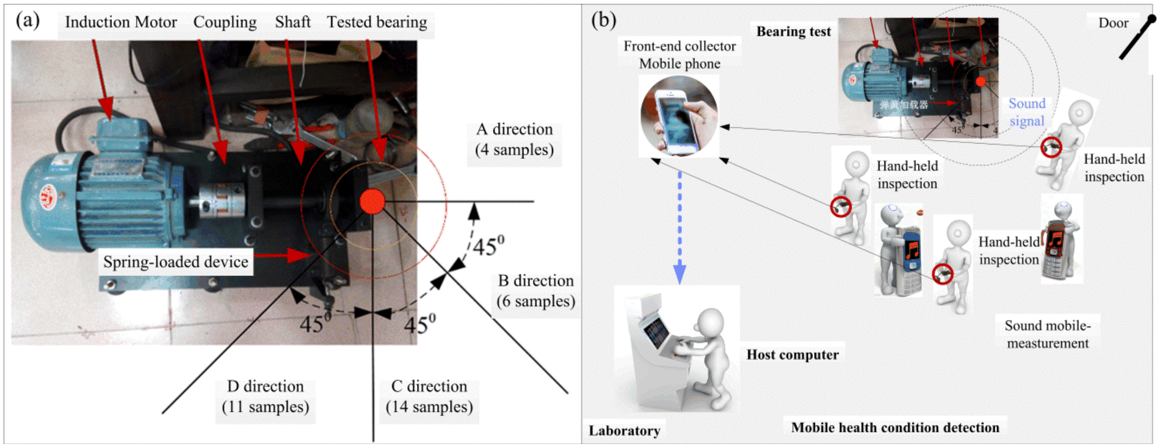

Figure 3. Experimental setup and mobile measurement for acoustic signal acquisition of ball bearing with outer-race defect: (a) bearing test, and (b) sound mobile-measurement.

Table 1: The conditions of the mobile measurement on the bearing test with the ZTE mobile phone in an android system (Data A)

Condition	Healthy	Outer-race fault
Measuring tool	An android mobile phone with a developed acquisition Software	
Measuring direction	Four direction (A_d, B_d, C_d, D_d) with 45 degrees interval	
Measuring samples	A_d-4 samples; B_d-6 samples; C_d-14 samples; D_d -11 samples	
Sampling condition	Sampling frequency 44.1 kHz with a time sampling 10s	

Table 2: The conditions of the mobile measurement on the bearing test with different measuring tools and locations (Data B)

Condition	Healthy		Outer-race fault	
Measuring tool	ZTE 889D (Person P)	MI 1 (Person Q)	ZTE 889D (Person P)	MI 1 (Person Q)
Measuring samples	25	28	56	50
Label	H-Z	H-M	F-Z	F-M

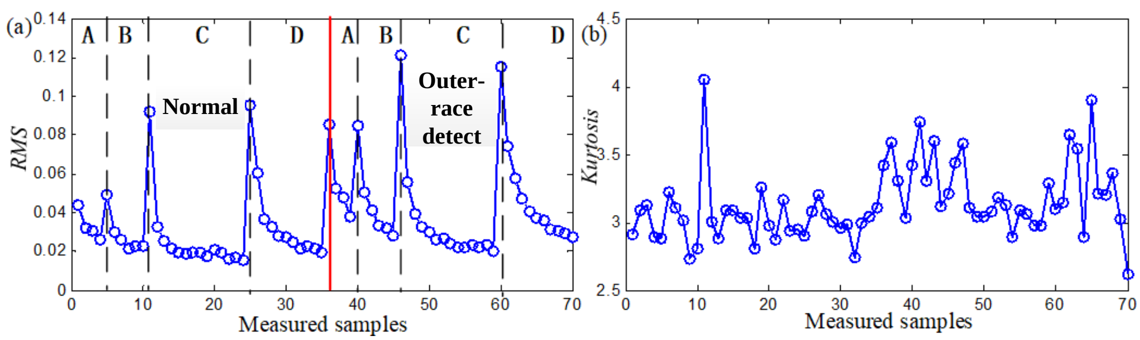

Figure 4. The health distribution of the measured Data A based on different HIs for Data A: (a) RMS and (b) Kurtosis.

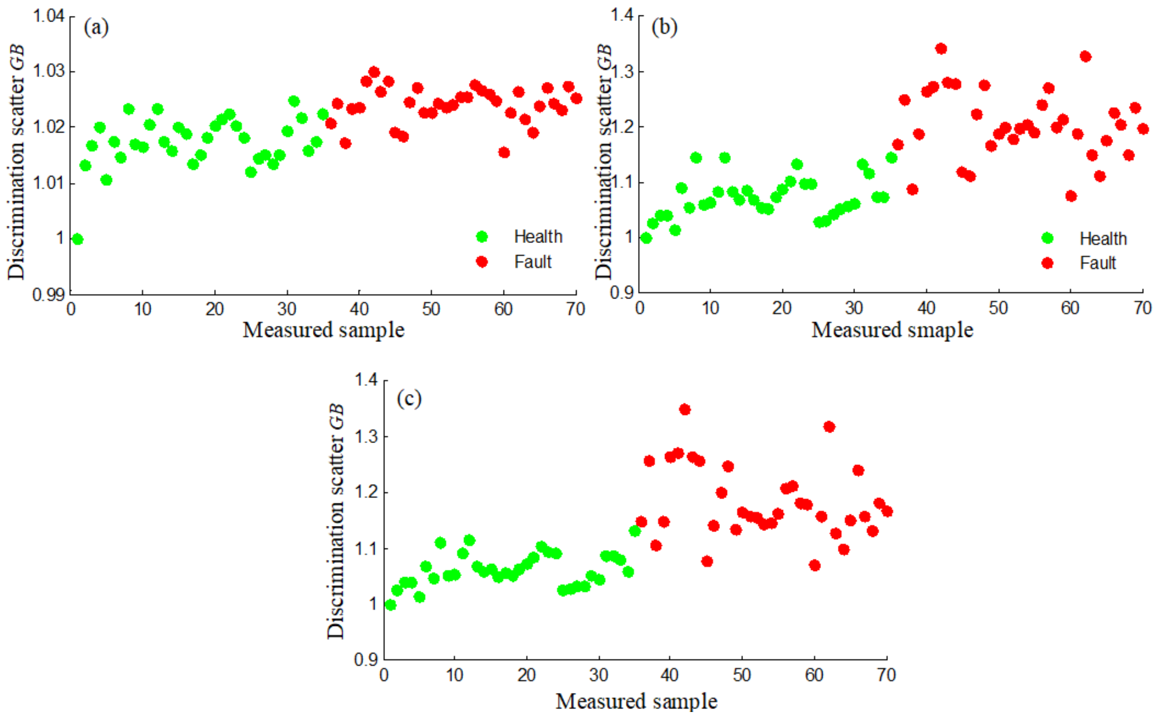

Figure 5. The health distribution of the measured Data A based on the proposed HI_{GM} with different kernel operation: (a) *GB*-Binet-Cauchy kernel, (b) *GP*-projection kernel and (b) *GC*-correction pseudo kernel.

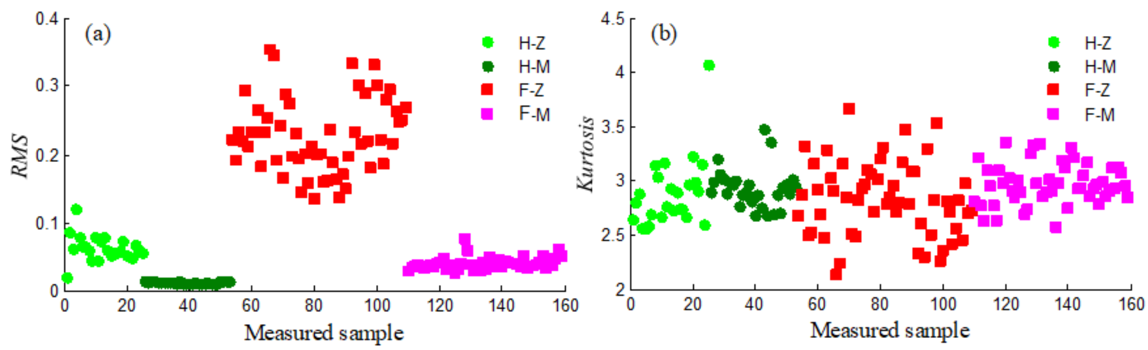

Figure 6. The health distribution of the measured Data B based on different HIs: (a) RMS and (b) Kurtosis.

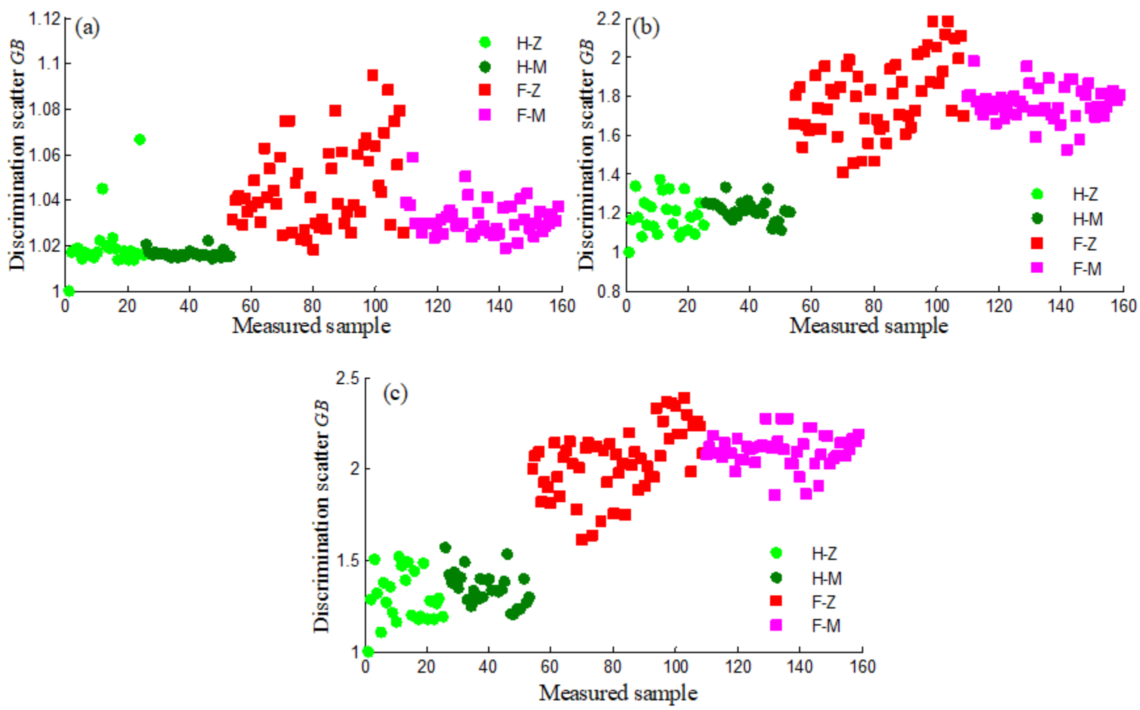

Figure 7. The health distribution of the measured Data B based on the proposed HI_{GM} with different kernel operation: (a) *GB*-Binet-Cauchy kernel, (b) *GP*-projection kernel and (b) *GC*-correction pseudo kernel.

Case II: To verify the effectiveness of the proposed method with the subspace clustering chart using a reference model, the study tests a case of a bearing running to failure. The experimental data are downloaded from Prognostics Center Excellence (PCE) through a prognostic data repository contributed by Intelligent Maintenance System (IMS), University of Cincinnati [21]. As plotted in Fig. 8, this study uses bearing 2 (2#) and bearing 4 (4#) of test 2 with outer-race failure to illustrate the performance of the proposed subspace clustering chart. The healthy data size n_0 and the monitored data size n_t are both set at 100, with a sample length of 1024. The size of the Hankel matrix is 20×20, and five principal bases are extracted to support the manifold basis space S; the reference model with the healthy subspaces and the monitored subspaces is at monitoring time t (t=1,2,…,T) is 200. It should be noted that the same healthy basis subspace is used in each monitoring process. Then, according to the spatial clustering analysis in GM space, as expressed in Eqs. (6)~(11), the GM-based discriminant factor G with three manifold kernel operations is calculated from the reference models to assess the performance degradation of the monitored bearing at each corresponding moment t.

For bearing 2#, as displayed in Fig. 9(a), the HIs based on *GB* and *GP* reveal a much clearer degradation trend; a significant and sharp change in the subspace clustering chart will occur once the monitored bearing degenerates from healthy status to slight degradation. Generally, these changing points can be used to accurately verify the occurrence of faults in a timely manner, helping to identify degradation at an early stage. An early alarm can be sounded, and effective maintenance can be arranged to restore bearing health. There is also a local fluctuation between the HI values, however, and this could have a bad effect on assessment.

In order to strengthen the sensitivity of HI to precisely assess the bearing performance degradation and detect the occurrence of degradation at an early stage, the study tests two traditional quality-control techniques: the exponentially weighted moving average (EWMA) and cumulative sum statistical control (CUSUM). These two techniques can be applied to the proposed GM-based subspace HI for a developed assessment chart. The EWMA-based HI can be defined as:

$$s_t = (1-\alpha)G_{t-1} + \alpha G_t \quad \alpha \in [0,1] \tag{12}$$

where a larger weight value of α allots more weight to current data G and less weight to older data. Unlike the consideration of the current status shown in Fig. 9(a), as displayed in Fig. 9(b), EWMA can associate the current status with the previous status, thereby achieving a much more sensitive response for PDA. It can detect a defect, and the degradation effect can be assessed in a clearer band. Furthermore, degradation is an irreversible physical process, and the status of the bearing will degrade with working time; these factors should be taken into consideration in the degraded HI. Accordingly, the CUSUM-based HI is calculated as:

$$c_i = \mathrm{CUSUM}(G_i - \mathrm{T}_{\mathrm{arget}}) \tag{13}$$

where $\mathrm{T}_{\mathrm{arget}}$ represents the average level of the health status in the past. This HI can indicate the cumulative deviation of the monitored bearing. As shown in Fig. 9(c), this improved subspace clustering HI can reveal a much more stable and clearer degradation trend with good monotonicity. It also detects different degradation rates, from slight degradation to severe degradation; this illustrates that the degradation of the full lifecycle is nonlinear.

The conclusion is shown in Fig. 10. The GM-based subspace clustering chart can accurately assess the degradation of a monotonic process, but the determination of degeneration based on the traditional HI is inaccurate, resulting in a confusing warning and incorrect diagnosis. Specifically, compared to the conventional HIs as displayed in Figs. 9 and 10, the proposed subspace clustering charts and their improvements, i.e., the HIs based on *GB* and *GP* kernel operation, can automatically reach a more accurate and sensitive PDA from the raw data by means of the GM-based spatial clustering analysis, and a significant warning will be clearly given. Therefore, spatial clustering analysis can provide a more accurate and comprehensible HI in real-word applications of bearing health monitoring.

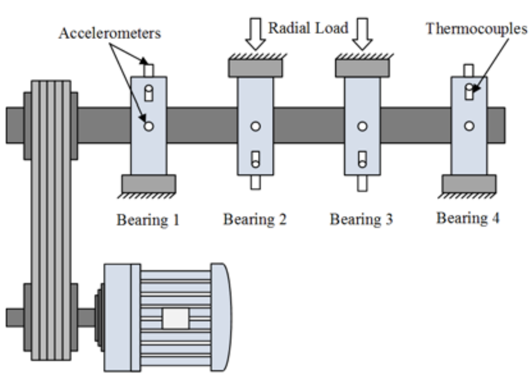

Figure 8. Bearing test rig

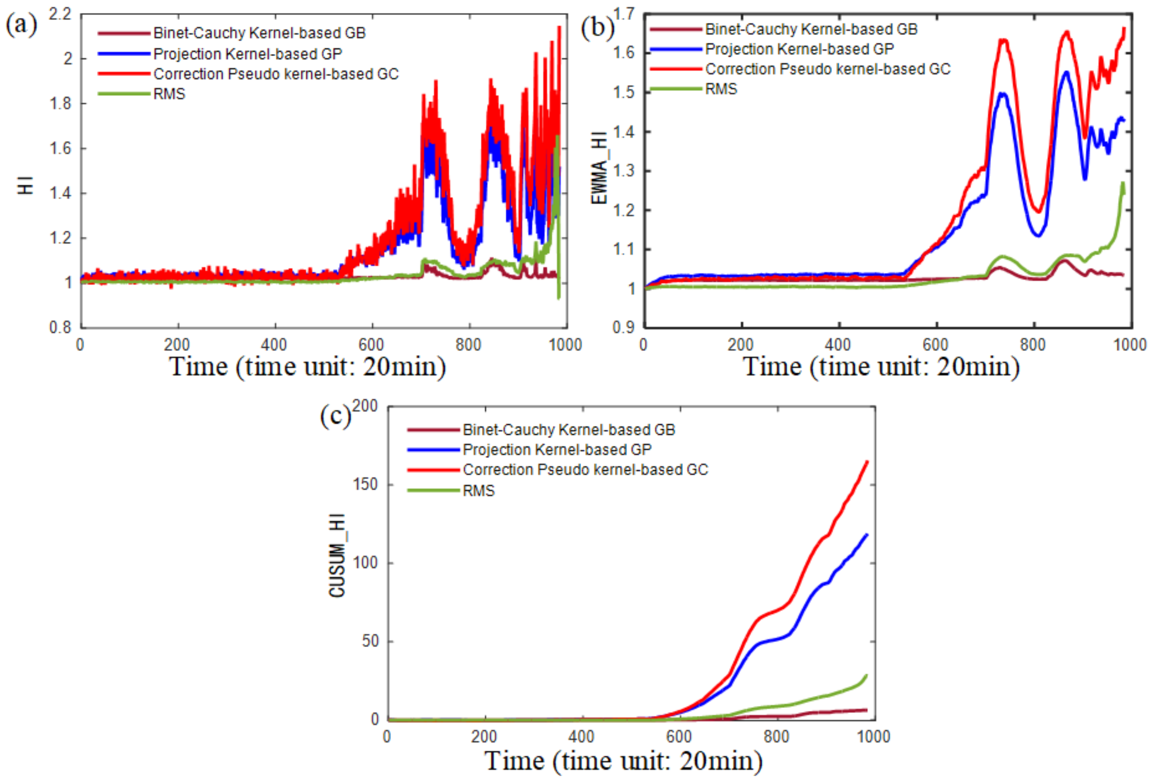

Figure 9. The PDA results for the full cycle life of the tested bearing 2#: (a) the raw HI, (b) the EWMA-based HI_w and (c) the CUSUM-based HI_c (the brown, blue, red and green lines represent different His, GB, GP, GC and RMS, respectively).

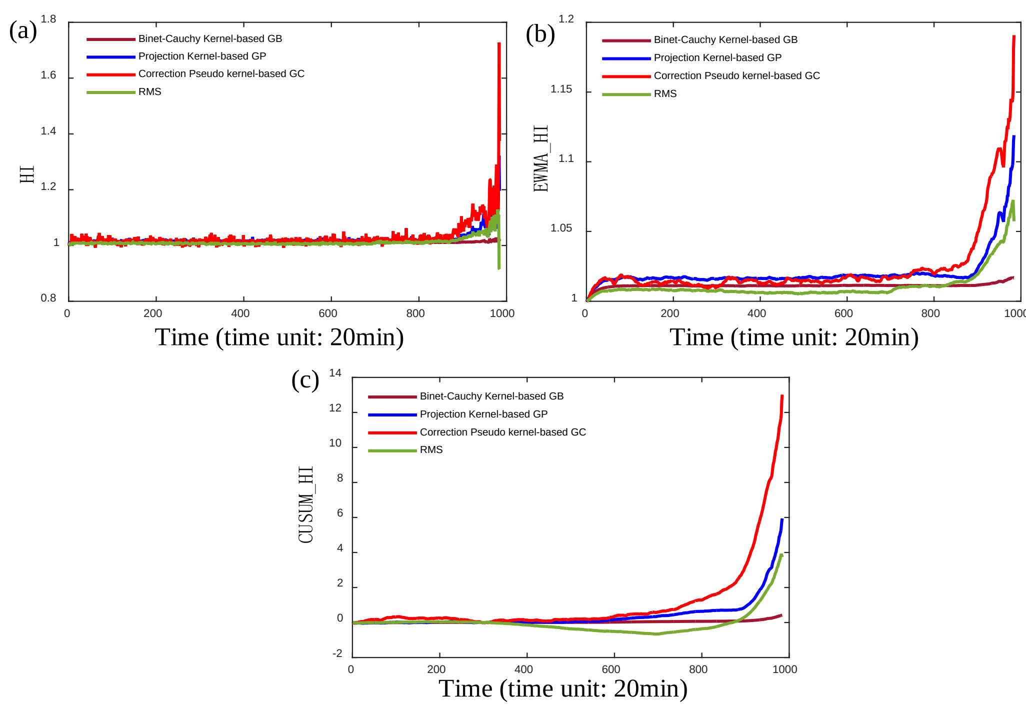

Figure 10. The PDA results for the full cycle life of the tested bearing 4#: (a) the raw HI, (b) the EWMA-based HI_w and (c) the CUSUM-based HI_c (the brown, blue, red and green lines represent different HIs, GB, GP, GC, and RMS, respectively).

Conclusion: The paper proposes the use of GM-based spatial clustering analysis to assess bearing performance degradation using the subspace clustering distribution of a two-class model to automatically depict the bearing's health condition. First, a covariance-driven Hankel matrix is constructed from the raw time-domain signal; manifold learning is then used to separate the feature subspace and noise null-space. This meaningful feature basis space can automatically represent the health conditions. Second, a reference model is built by combining the reference healthy data subspaces and the monitored subspaces. A new subspace clustering HI with kernel operation is implemented to assess the current machine performance, revealing the features that discriminate abnormality from healthy operation. Subspace clustering HI is used to experimentally detect abnormal bearing condition using the mobile mapping mode. The study finds that the spatial clustering analysis using the reference model is effective to detect abnormalities in a self-driven way. Finally, a novel subspace clustering chart, CUSUM-based HI, is developed to depict the real bearing performance degradation assessment (PDA).

Compared to the regular HI, (e.g., root mean square, kurtosis), the proposed approach can provide a more accurate and reliable degradation assessment profile with an early fault occurrence alarm. The case study comparisons show the potential of the proposed spatial clustering analysis to assess bearing health performance degradation. For an extension of the GM-based spatial clustering, this reference model can be further implemented in the PDA of other key machine components and in remaining useful life (RUL) predictions.

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