

# PRACTICAL ROTOR DYNAMICS FOR USERS

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## **Abstract:**

Following a brief overview of the history of the development of key aspects of rotor dynamic technology, this paper describes ways to estimate critical speeds, and cipher them from resonances and other vibration phenomena. The influence of various bearing types on a rotor's synchronous and non-synchronous dynamics is reviewed. Target audiences for this material are those most likely to be making and interpreting vibration measurements - in other words, machinery users and maintainers.

**Key words:** Bearings; critical speed maps; damping; diagnostics; health management; prognostics; rotor dynamics; vibration assessment

**Introduction:** In many cases, the in-field vibration analyst has minimal design information (shaft dimensions, bearing properties, predicted critical speeds, damping, etc.) regarding the machine being assessed. Nonetheless, s/he must make determinations regarding the “health” of the equipment and in many cases, recommend and/or implement a “fix” if the machine behavior is changing or faulty.

This paper recalls some of the things the in-field analyst might consider to help determine a machine’s operational condition. The intention of this short review is to provide insight/memory jogs regarding test methods to acquire information that can help get the job done. Technology advances in the past 50 years make the work more accurate and easier.

**The Past Enables the Present:** Acknowledging this conference’s theme “Fifty years of failure prevention technology innovation,” this section summarizes some of the enabling advances that benefit today’s rotor dynamic technology. In actuality, some developments have been more than 50 years in the making, as described below. Looking back, advancements have closely paralleled the progress made in digital computing; principally increases in computing power and decreases in cost and size.

Analysis Methods: The study of a rotor’s dynamic behavior began in the late 1800’s (1-3). Early theoretical and experimental contributors included William J. M. Rankine (1869 - first performed an analysis of a spinning shaft), Gustaf de Laval (1889 - ran a steam turbine to supercritical speeds), and Stanley Dunkerley (1895 - published an experimental paper describing supercritical speeds). In 1916, W. Kerr published a paper showing experimental evidence of a second critical speed, and in 1919 Henry Jeffcott

published a paper now considered classic in which he confirmed the existence of stable supercritical speeds. History suggests August Föppl in 1895 described work similar to Jeffcott's, but it was unfortunately published in a German civil engineering journal that went unnoticed by the rotating machinery technical community at the time.

Through the mid 1900's, much work was done in the area of instabilities and modeling techniques culminating in the works of H. Holzer, Nils Myklestad and M. A. Prohl which led to the transfer matrix method (TMM) for analyzing rotors. The approach involves assumptions and variation of a frequency until the boundary conditions are met. The most prevalent analysis technique used today is the finite element method (FEM), and there are many PC-based programs commercially available. The FEM method assumes a mode shape in equations of motion that meet the boundary conditions and iterates the assumed mode shape until it does not change, thus obtaining a frequency that relates to the calculated mode shape.

In parallel with this activity, groundbreaking work was being accomplished to assess the dynamic performance of bearings, and the resulting development of bearing dynamic properties that are used in rotor dynamic computer programs (4-7). Much of the early work focused on fluid film bearings, the principal style used in ever-larger turbomachinery being developed at the time.

Sensors: Of the three principal ways to assess machinery vibration (displacement, velocity and acceleration), in the author's opinion it is the development of the eddy current displacement sensor that over the past 50 years has had the most important role in development of rotor dynamic technology. The ability to directly and accurately measure shaft motion without contact over wide speed ranges has provided practitioners with information needed to verify or dispute predicted performance. Among the early developers was Donald E. Bently (1924-2012). In today's marketplace, eddy current "proximity" sensors are available as commodities from multiple suppliers.

Instrumentation: From a field measurement perspective, this is where the most obvious advances have occurred. Improvements have enabled easier and more accurate acquisition and interpretation of rotor dynamic data. Fifty years ago, vibration instruments were analog devices, and the input signal was passed through filters of various widths. Analyzers were large, bulky, heavy devices that were called portable if their cases had handles! Recording devices were multichannel analog tape recorders that were even bigger and weightier. To enable machinery reliable enough for months-long voyages and quick refits during crew changeovers, the U.S. Navy submarine fleet spurred development of crew-usable equipment, and had the most comprehensive machinery vibration recording equipment and onboard machinery health monitoring methodology. The digital revolution enabled evolution to minicomputers, microcomputers, personal computers to today's hand held devices. Dynamic range for these instruments has increased from 30-40 dB to 120 dB or more, enabling easier and far more accurate and concise data acquisition and processing.

**Using Transient Vibration to Assess Rotor Dynamics:** For the in-field vibration analyst, measuring a machine's transient vibration provides a wealth of rotor dynamic information. Benefiting from the type of advances outlined in the previous section, these measurements can be accomplished more easily and with better accuracy than ever. Available from many text and vibration books and course notes, information in this section has been principally extracted from references 2 and 8.

Observing transient vibration can help identify natural frequencies, confirm system resonances, obtain damping values, and obtain modal information if needed to correction of resonance. There are several ways to obtain the data needed. One of these is illustrated below. Other methods (9-12) include measurements made as machine speed sweeps through a rotor critical speed, the half power method, and the phase angle method.

This example uses the shaft's lateral motion (ring-down) to a transient event, perhaps an impact on the machine's housing. The machine may be operating or non-operating during these measurements. Depending on the style of bearings the machine has (see a following section), results may be improved if the machine is running. In most instances, structural and other non-rotor-bearing resonances may not be too noticeable in ring-down tests. One must of course be aware of any sensor mounting resonances that may be encountered, since shaft observing sensors will observe those as well.

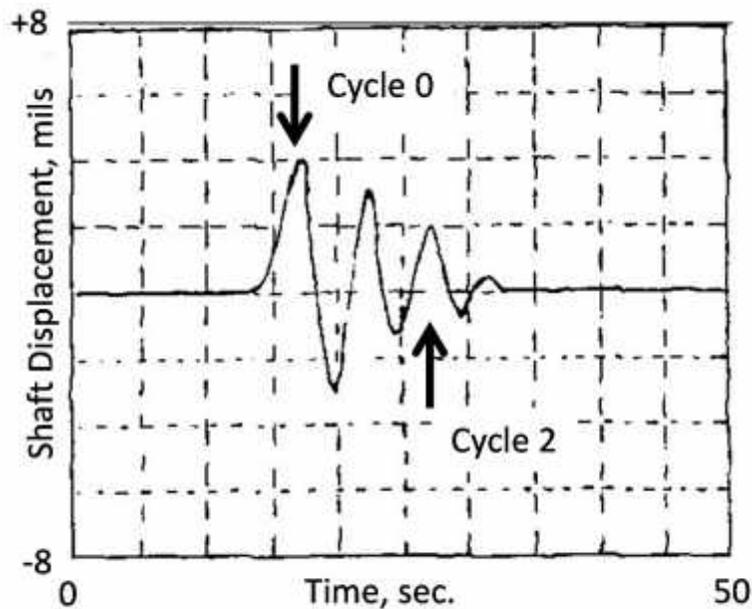


Figure 1: Rotor Response to Impact

Classical derivations (9, 10) provide the following forms for logarithmic decrement (used to determine system damping from measurements), damping ratio, amplification factor, period of vibration, and natural frequency:

$$\text{Logdec} = u = \frac{1}{n} \text{Ln} \frac{X_0}{X_n} = \frac{1}{2} \text{Ln} \frac{4}{2} = 0.3466 \quad (1)$$

$$\zeta = \frac{1}{2f} u = \frac{.3466}{6.28} = 0.055 \quad (2)$$

$$Q = \frac{1}{2\zeta} = \frac{1}{2 * 0.055} = 9.1 \quad (3)$$

$$T_d = 5 \text{ msec} = 0.005 \text{ sec} \quad (4)$$

$$f_d = \frac{1 \text{ cycle}}{.005 \text{ sec}} = 200 \text{ Hz} \quad (5)$$

Based on the above equations and Figure 1 data, the logarithmic decrement is 0.3466 (equation 1). The damping ratio (2) is 0.055 a small but oftentimes sufficient amount. The amplification factor (3) is 9.1, a tolerable value in many machines. The measured period (4) is 0.005 seconds and the natural frequency (5) is 200 Hz.

**Forced Harmonic Vibrations – A Key to Understanding Rotor Dynamics:** Harmonic vibration is found in most real world machines, with mass unbalance (see a following section) being the most common. Although other types of excitation are always present, knowing how the system responds to harmonic excitation provides insight into more complex behavior too. Most of the material presented in this section comes from reference 2.

Based on classical derivations, Figure 2 presents the motion of a machine which can be modeled as a single degree of freedom system. The Magnification Factor depicts the factor by which the zero frequency deflection must be multiplied to determine the amplitude X, based on how near or far the frequency is from the natural frequency, where  $\frac{\omega}{\omega_n} = 1$ .

The figure shows that amplitude and phase in the vicinity of a natural frequency are functions only of the frequency ratio and the damping factor. As shown, damping has a large influence on amplitude and phase in this region, but little or no influence elsewhere. In other words, damping is important only when the forcing frequency is close to the natural frequency.

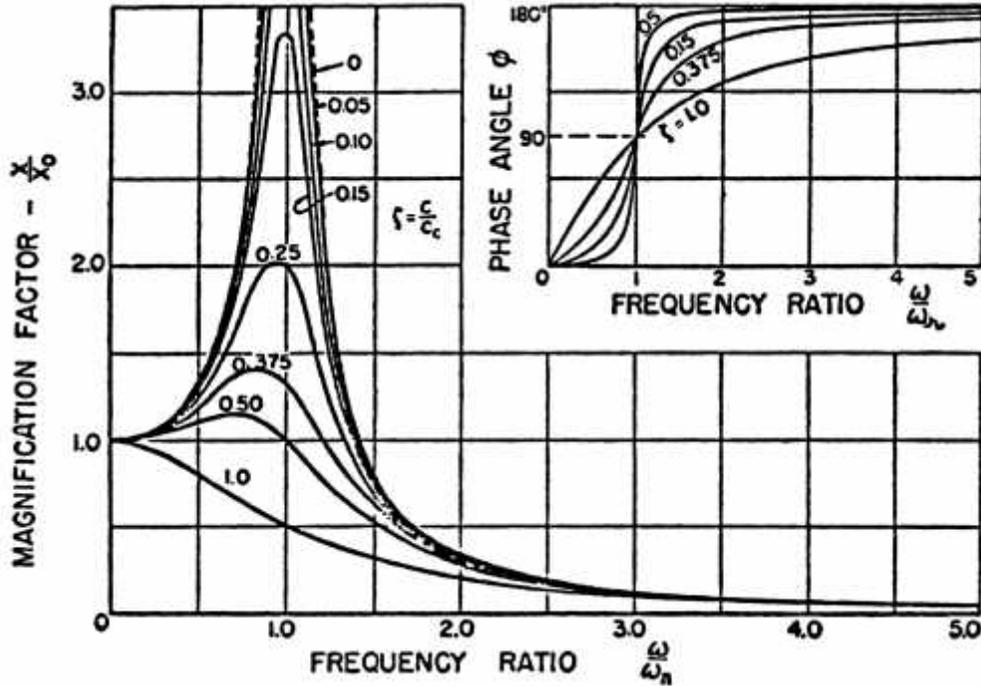


Figure 2: Forced Vibration of a Viscously Damped System

**Shafts and Rotors:** It is important for in-field vibration analysts to understand the concept of rigid and flexible rotors. Figures 3 and 4 show some basic concepts that lead to the critical speed map of Figure 5, the key take-away from this section. Information in this section was principally extracted from references 2 and 11.

A rotor consists of a shaft with an assembly of disks mounted on it (Figure 3a) or a massive shaft with varied changes in geometry (Figure 3b). A rigid rotor operates substantially below its first bending critical speed. In other words, its flexural vibration is inconsequential compared to bouncing and rocking on the bearings. A rigid rotor can be balanced at an arbitrary speed in two planes and remain in balance at all operating speeds. A flexible rotor, Figure 4, operates close to or beyond its first bending critical speed and dynamic effects influence relative rotor deformation.

Rotor flexibility depends on the relative stiffness between the bearings and the rotor and its operating speed. Figure 5 shows a critical speed map. This map shows the effect of the relative flexibility of the rotor and bearings on system natural frequencies.

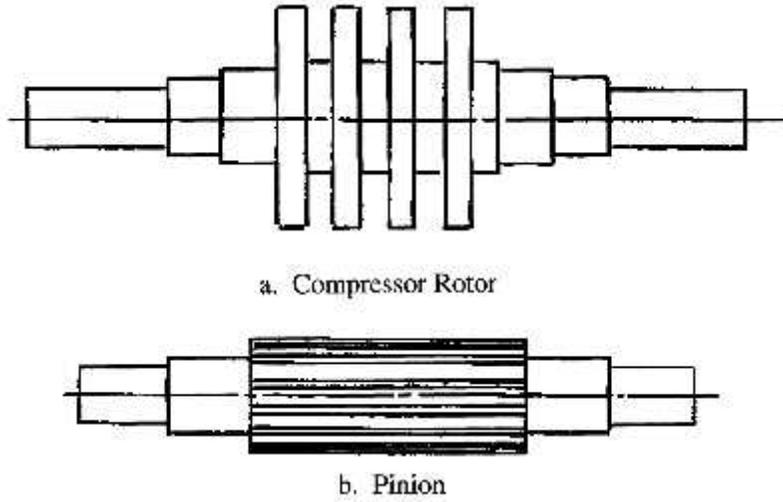


Figure 3: Rotor Geometry

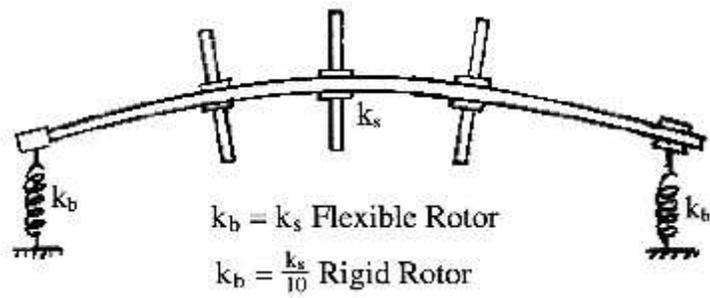


Figure 4: Rotor-Bearing Flexibility

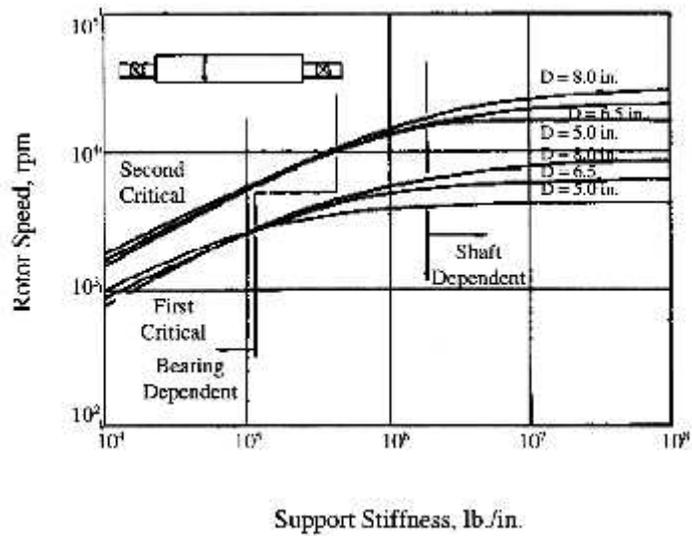


Figure 5: Critical Speed Map Showing Rigid and Flexible Shaft Sectors (11)

**Mass Unbalance Cause a Principal Force in Machinery:** The classical mass unbalance effects result in a once-per-revolution frequency with amplitude proportional to the rotor mass, mass eccentricity, and rotor speed squared (equation 6). Most of the material presented in this section comes from references 2 and 12.

$$F_c = \frac{w}{g} \omega^2 a \quad (6)$$

Where  
 a = eccentricity, in.  
 w = weight, lb.  
 $\omega$  = speed, rad./sec.  
 g = gravitational constant, in./sec.

Mass unbalance arises from the lack of coincidence of the mass center with the spin axis of the rotor (Figure 6). As it is accelerated, this mass eccentricity causes a centrifugal force on the rotor. The force due to mass unbalance increases as the speed squared. In addition, it can be seen (Figure 2) that the amplitude will be a function of shaft and support stiffness, damping, and mass; i.e., the natural frequency (critical speed). Extracted from reference 12, typical causes of mass unbalance are shown in Table 1.

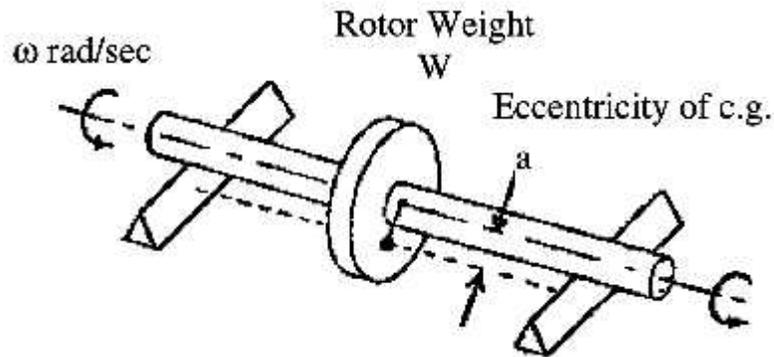


Figure 6: Variation of a rotor's Mass Center

Table 1: Typical Causes of Rotor Unbalance

CAUSES OF UNBALANCE	OBSERVABLE SIGNS	FREQUENCY OF VIBRATION
Disk or component eccentric on shaft	Detectable runout on slow rotation, e.g. runs to bottom on knife edges	one/rev
Dimensional inaccuracies	Measurable lack of symmetry	one/rev
Eccentric machining or forming inaccuracies	Detectable runout	one/rev
Oblique angled component	Detectable angular runout Measure with dial gage on knife edges	one/rev
Bent shaft Distorted assembly Stress relaxation with time	Detectable runout on slow rotation, often heavy vibration during rotation	one/rev
Section of blade or vane broken off	Observable bearing vibration during operation	one/rev; possible process pulsations
Eccentric accumulation of process dirt on blade	Bearing vibration	one/rev
Differential thermal expansion	Shaft bends and throws c.g. out. Source of heavy vibration	one/rev
Non-homogeneous component structure Sub-surface voids in casting	Rotor machined concentric Bearing vibration during operation. C.G. runs to bottom on knife edges	one/rev
Non-uniform process erosion	Bearing vibration	one/rev
Loose bolt or component slip	Vibration reappears after balancing due to component angular movement	one/rev Possible magnitude and phase changes
Trapped fluid inside rotor, possible condensing/ vaporizing with process cycle	Vibration reappears after balancing Apparent c.g. angular movement occurs	one/rev Possible magnitude and phase changes
Ball bearing wear	Bearing vibration Eccentric orbit with possible multi-loops	one, two, or higher per rev

**Bearings & Their Effect on Rotor Dynamics:** A critical speed map (Figure 5) shows how the effective stiffness and damping of a machine's suspension affects its vibration overall. This section briefly describes the principal styles of machinery bearings and compares their relative attributes.

- **Rolling Element Bearings:** These bearings come in a wide variety of styles and sizes, including Deep Groove, Angular Contact, Four-point Contact, Self-aligning, and even Linear Sleeve and Linear Spline. Rolling element bearings are used in wide speed ranges from low to very high. Compared to other bearings, they have low friction (they are sometimes called antifriction bearings!), high stiffness, and low damping.
- **Plain Bearings:** These are the simplest of bearings. They involve just two surfaces: one moving, one stationary. They are often compact, lightweight, and inexpensive. Nonetheless, they are capable of very high loads. Two general categories include Integral, where the bearings are built into an object, and Bushing, where the bearing is an independent object that is inserted into a housing. Plain bearings use one of three categories of lubrication systems: Class I – lubricant applied from an external source; Class II – lubricant contained within the bearing walls; Class III – the bearing material is the lubricant.
- **Fluid Bearings:** For these bearings, fluid lubrication separates the two surfaces through fluid dynamic effects. The fluid can be liquid or gas. There are two categories of fluid bearings: Hydrostatic bearings, where external pressure provides the load capacity, and hydrodynamic bearings that use the relative motion between surfaces to generate the load carrying capacity. Hydrostatic bearings are often used when machinery speeds are low. Hydrodynamic bearings are more suitable for higher speeds. Gas lubrication is better suited to light loads and high speeds with lower losses, while liquid lube enables higher loads and more effective cooling.
- **Magnetic Bearings:** These bearings support machinery without physical contact. They can be used over a wide speed range, and have very low friction, and no mechanical wear. Compared to rolling element or oil lubed bearings, magnetic bearings have lower stiffness. There are two categories of magnetic bearings: Permanent magnet (PM), which are passive and use the repulsive nature of north and south pole magnets, and electromagnet (EM), which can carry higher loads but need shaft position sensors and a controller that adjusts magnetic field strength on-the-fly. PM bearings have low stiffness and little damping. EM bearings can include sophisticated controllers that adjust the bearing's effective stiffness and damping. Auxiliary bearings for overload or controller power loss are often needed for EM bearings.

**Conclusions:** Although the in-field vibration analyst's access to design-related background information is often limited, proper selection and application of sensors, instrumentation, and some straightforward tests can provide meaningful insight towards understanding the vibration of the machine at hand. It is important to understand that the machine's rotors and bearings form a dynamic system, and that "fixes" for one type of machine may not apply universally. The capabilities available to assist today's practitioner build upon the developments of many theorists and practitioners in decades past.

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