RELIABILITY MODELING OF COMPLEX MULTI - STATE SYSTEM BASED ON BAYESIAN NETWORKS

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Abstract: Traditional reliability analysis methods are not suitable for some systes with multi-state probability. In this paper, a Bayesian Network based multi-state reliability model is built according to the state relationship between the system and its constituting components. Universal generating function is used to build the conditional probability table of the non-root nodes. Based on this, a method of computing the system failure state's probability is proposed. The algorithm of the importance of each component in the system is inferred. Combined with an example of a typical natural gas pipeline compressor zone system is given. The result shows that this method in this paper provides a simple and intuitive measure to deal with the reliability analysis of natural gas pipeline compressor zone system with multi-state property. The reliability level can be evaluated and the affection of each component to the system reliability can be confirmed effectively.

Key words: Multi-state reliability; universal generating function; Bayesian Network

1 Introduction: Traditional reliability analysis is based on the assumption that events are binary, i.e., success and completely failure, while in actual engineering, many systems are in a state of partial failure between normal and complete failure, these states should be taken into account, especially when more accurate assessment of system reliability and system optimization design are necessary. Thus, the traditional re-liability theory of two states system can't meet these requirements; Multi-State System (MSS) reliability analysis is very important and essential.

MSS reliability analysis has received extensive attention over the past years; many researchers have put forward many methods in this field [1]. Because of the similarity between the MSS and traditional two states (or failure) system in many ways, the traditional modeling and analysis methods are extended and expanded in the field of MSS [2-3]. The modeling and analysis methods of multi-state fault tree are studied, based on the extension of traditional Binary Decision Diagram (BDD), a multi - state binary decision diagram (MBDD), a binary decision diagram (LBDD) of logarithmic transformation and a Multi - state binary decision diagram Multi-valued decision diagram MMDD, etc., are formed, which have been widely used. But the expression and solution method of the block diagram in the multi-state system reliability is relatively small; Markov analysis has the defects of state space explosion and cannot be used to model the complex and bulky systems.

In this paper, a new method for reliability modeling of complex multi-state systems based on Bayesian networks (BN) is proposed. The nodes of the BN and the various states of the components of the sys-tem are determined, and the probability of each state is given [4]. Then, conditional probability distributing (CPD) is used to describe the relations among the states of the elements to express the state of the associated nodes. Finally, a BN model of multi-state complex system is established. The model can express the state and state probability of the system and components clearly, and can directly calculate the system reliability according to the multiple state probability of the component, and qualitatively analyze and quantitatively evaluate the reliability of the multi-state system. In this paper, the marine steam turbine generator system is used as an example to verify the model, and the Bayesian network inference is used to analyze the reliability and component importance of turbo-generator system.

2 Introduction of multi-state system and Bayesian Network:

2.1 Multi-state systems: As the name suggests, a multi-state system is a system with multiple operating states, these different states can be different in performance, for example, the generator set can work at different power levels; can also be different in failure states, such as system with the long-time operation, due to wear and tear, fatigue and other failure modes lead to the gradual degradation of system performance, making the sys-tem to run a different level of degradation; can also be a system with different failure modes, which has different effects on the upper level system or other, such as the two failure modes of the valve. To sum up, a multi-state system is a system with a limited number of performance levels, and its various sub-systems and components are generally multi-state.

In the context of this study, the definition of the working state of the system or component is a certain state which can achieve a certain level of performance requirements or complete certain tasks, as well, the failure state of system or component which is unable to achieve a certain level of performance requirements or to complete certain tasks.

To analyze the reliability of a multi-state system, it is first necessary to analyze the characteristics of its components. Assume that any component j of the system has k_j

kinds of different states without loss of generality, corresponding to the performance level of the component in different states, which can be represented by a set as follow:

$$\boldsymbol{g}_j = \left\{g_{j0}, g_{j1}, \cdots, g_{jk_j-1}\right\} \tag{1}$$

Where g_{ji} represents the performance level of component which is in the *i*th state, $i=0,1,\ldots,k_j-1$.

The performance level G_j of component j at any time is a random variable, which value range is determined by $\boldsymbol{g}_j$, $G_j \in g_j$. In some cases, the performance level G_j of component j cannot be determined by a single random variable, but may be determined by the random variables of several performance parameters, so the performance level at this time should be defined as random vector $\boldsymbol{G}_j$. In the multi-state system discussed in this paper, this situation is not considered.

The state probability of component j at different performance levels can be expressed as a set:

$$\boldsymbol{p}_j = \left\{p_{j0}, p_{j1}, \cdots, p_{jk_j-1}\right\} \tag{2}$$

Where $p_{ji} = \Pr\left\{G_j = g_{ji}\right\}$.

Similar to the definition in a two-state system, the state probability of a component of a multi-state system is the probability of being in a state at a particular task time, and it is required that these state events be mutually exclusive, i.e. component j can only be in one state, and the sum of the probabilities for each state is 1.

Equation (2) defines the probabilistic quality function of the random variable G_j, i.e. the set of performance level states and their combinations of probability (g_{ji}, p_{ji}) can form the distribution of the performance level of the component. The cumulative distribution is shown in Figure 1. The horizontal axis x represents the performance levels of the component, and the vertical axis represents the probability that the component's performance level is not less than x.

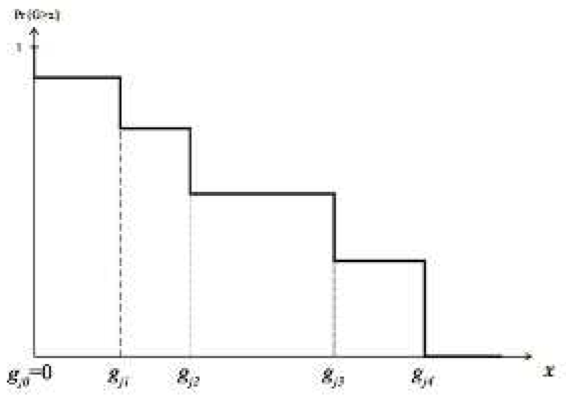

Figure 1: Cumulative Performance Distribution Curve of Multi - State System Components

For a multi-state system with n components, the performance level of the system is determined by the performance level of each component. Assuming that the whole system has K different states and g_i represents the performance level of the system at the ith state $(i \in \{0,1,\cdots,K-1\})$, then the performance level of the system is a random variable whose value range can form a set as$\{g_0,g_1, \cdots,g_{K-1}\}$.

A model of a multi-state system is defined, which includes the probability-quality function of the performance levels of the various system components (Eq. (3)) and the structural function of the system (Eq. (4)).

$$g_j, p_j, 1 \le j \le n \tag{3}$$

$$\varphi(G_1, K, G_n) \tag{4}$$

The universal generating function is used to describe the serial-parallel structure of multi-state component or system, and then the generating function polynomial of multi-state element is [5-7]:

$$U_j(z) = \sum_{i=0}^{K_j-1} p_{j_i} z^{g_j} \tag{5}$$

Where, z is an auxiliary variable, this formula also applies to multi-state systems.

1) Assuming n components in series connection, the generating function of the system is:

$$U_j(z) = m \quad (U_0(z), U_1(z), \dots, U_{n-1}(z)) = \sum_{i_0=0}^{K_0-1} \sum_{i_1=0}^{K_1-1} \dots \sum_{i_n=0}^{K_n-1} \prod_{j=0}^{K_n-1} p_{j i_j} z^{m \quad (g_{0 j_0}, g_{1 j_1}, \dots, g_{n-1 j_{n-1}})} \tag{6}$$

2) Assuming n components connected in parallel, the generating function of the system is:

$$U_j(z) = m \quad (U_0(z), U_1(z), \dots, U_{n-1}(z)) = \sum_{i_0=0}^{K_0-1} \sum_{i_1=0}^{K_1-1} \dots \sum_{i_n=0}^{K_n-1} \prod_{j=0}^{K_n-1} p_{j i_j} z^{m \quad (g_{0 j_0}, g_{1 j_1}, \dots, g_{n-1 j_{n-1}})} \tag{7}$$

2.2 Bayesian Network: Bayesian network (BN), also known as Belief Network, is a directed acyclic graph (DAG) with probability annotated [8]. Considering a finite discrete random variable set $U=V_1,V_2,\cdots,V_K$, where, each variable V_i can take a finite number of values. Formally, a Bayesian network is a two tuple $S=<G, P>$. Of these, the first part G is a directed acyclic graph where the nodes mirror the random variables $V_1,V_2,\cdots,V_K$, and directed arch shows conditional dependencies between variables. It contains the following conditional independence assumption: given set of parent nodes, each variable is independent on its non-descendant nodes. The second part P is a set of parameters that describe the network distribution of the conditional probability and quantitatively represents dependence of each node with its parent nodes. Node without the parent node is called the root node; node without descendants is called a leaf node.

Based on the probability multiplication formula:

$$p(V_1, V_2, \ldots, V_k) = \prod_{i=1}^{k} p(V_i | V_1, V_2, \ldots, V_{i-1}) \tag{8}$$

If $A(V_i)$ represents any subset of nodes which non- V_i descendant nodes constitute, $Pa(V_i)$ represents direct parent nodes, then the assumption based on the conditional independence is:

$$p\big(V_i \big| A(V_i), Pa(V_i)\big) = p\big(V_i \big| Pa(V_i)\big) \tag{9}$$

Variable U joint probability distribution Bayesian network described can eventually be determined by the following formula only:

$$p(U) = p(V_1, V_2, \ldots, V_k) = \prod_{i=1}^{k} p\big(V_i \big| Pa(V_i)\big) \tag{10}$$

Bayesian network construction steps are as follows:

a) Determine the corresponding relationship between variables with each node of the network.
b) Constructing a directed acyclic graph indicating conditional independence.
c) Determine conditional probability distribution parameters of nodes.

Figure 2 is a simple yet typical Bayesian network model, the joint probability of the variables in the model can be deduced by equation (10):

$$p(V_1, V_2, V_3, V_4) = \prod_{i=1}^{4} p\big(V_i \big| Pa(V_i)\big) = p(V_4|V_2, V_3)p(V_3|V_1)p(V_2|V_1)p(V_1) \tag{11}$$

Bayesian network inference is realized by computing the probability of certain conditions, including the joint probability, the marginal probability as well as the calculation of each conditional probability.

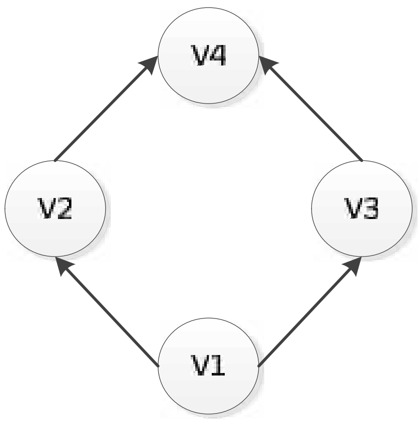

Figure 2: Bayesian network model (omitting the CPT)

Let the Evidence set of variables be E, Query be Q, under the condition of given evidence variable values $E = e$, Bayesian network inference calculates conditional probability distribution of the query variable Q. It can be described as follows:

$$p(Q|E = e) = \sum_{U-E} p(V_1, V_2, \ldots, V_k) = \sum_{U-E} \prod_{i=1}^{k} p\big(V_i \big| Pa(V_i)\big) \tag{12}$$

3 Multi-state systems reliability modeling:

3.1 The state probability of component: Before modeling the system by the Bayesian network, the state of all the basic components is analyzed and divided, and the state probability of each state should be solved. For different situations, different methods can be used to obtain the state probability of the system.

1. State probability of non - repairable system with known reliability function
Assuming that the reliability function of the component j which changes with time t is $R_j\left(t\right)$, so that the reliability value of the component at any time t_0 can be obtained, the state probability distribution of the component at time t_0 can be considered as shown in Table 1:

Table 1: State probability distribution of non - repairable two - state system

State X	0	1
Probability	1- $R_j\left(t_0\right)$	$R_j\left(t_0\right)$

2. The state probability of the degenerate component with known degradation behavior
In a multi-state system, many components progress gradually over time and their performance is gradually declining until they fail to meet the performance requirements of the system. For such components, the performance degradation analysis theory can be used to analyze the state of the component.

For the components whose performance level is represented by a single performance parameter, we can perform a degradation test on them, the test data are distributed and fitted to obtain the degraded failure distribution of the components. For highly reliable products, an accelerated degradation test can be performed to get the test data [9].

After the degradation failure of the component is obtained, a degenerate failure probability can be obtained by setting a performance level as a failure threshold for the two-state component. For multi-state components, it is necessary to discretize this distribution, specify some state of discrimination, and set the threshold of each state, which can be the status of each state probability.

3. Solving method of state probability based on Markov Process
According to the time is continuous or discrete, the Markov process is divided into continuous-time Markov process and discrete event Markov process.

The probability of its state can be estimated by the Markov chain when the probability of transition between the states of the component is known [10]. However, the general Markov process can only solve the case where the life distribution of the component is exponential distribution, so for the case of non-exponential distribution, the semi-Markov process can be used to solve. Firstly, the process of converting between states in multi-state components is expressed as a continuous-time discrete state stochastic process, and then the integral equation of state transition is given as shown in Eq. (13).

$$\phi_{ik}(t) = \delta_{ik}\left[1 - F_i(t)\right] + \sum_{l=1}^{m_j} \int_0^t q_{il}(\tau)\,\phi_{lk}(t-\tau)\,d\tau \tag{13}$$

Where

$$q_{il}(\tau) = \frac{dQ_{il}(\tau)}{d\tau},\ F_i(t) = \sum_{l=1}^{m_j} Q_{il}(\tau),\ \delta_{ik}(\tau) = \begin{cases} 1, & \mathrm{i} = \mathrm{k} \\ 0, & \mathrm{i} \neq \mathrm{k} \end{cases}$$

Then, after giving the kernel matrix of the semi-Markov process and the initial conditions, we can obtain the state probability of each state [11].

3.2 Calculation of system failure probability: According to the expression of Bayesian network inference, the joint probability distribution is the product of all probabilities in the network [12] [9], which is:

$$P(X_0, X_1, \ldots, X_{n-1}) = \prod_{k=0}^{n-1} P\left(X_k \middle| pa(X_k)\right) \tag{14}$$

Where, *pa* (X_k) is the parent node of X_k.

Therefore, in the Bayesian network, the joint probability distribution can be used to directly compute the probability of occurrence of the top event *T*, that is, the failure probability of the system:

$$P(T = 0) = \sum_{X_0, X_1, \ldots, X_i} P(X_0 = x_0, X_1 = x_1, \ldots, X_i = x_i, T = 0) \tag{15}$$

Where X_0, X_1, ..., X_i are the bottom and middle nodes in the network, x_i is the possible state value of event *i*, 0 is the complete failure of the event; *i* is the number of non-leaf nodes in the network. According to Eq. (7) and (8), we can calculate the probability of occurrence of the top event *T*, that is, the failure probability of the system.

3.3 Calculation of structural importance: Computing the structural importance of a component is an important part of the reliability analysis. It is used to reflect the degree of change of the system caused by the slight change of the state of a component [13-14] [10-11], and to find out the weak points of the system.

The structural importance degree *I (j)* of the element X_j can be calculated by the Bayesian network inference [15] [12], the equation as follows:

$$I(j) = \frac{1}{m-1}\sum_{l=0}^{m_j}\left[\left|P(T = 0|X_j = l) - P(T = 0|X_j = 1)\right|\right] \tag{16}$$

Where m_j represents the fault state of the element X_j, l indicates that the element X_j is in the fault state, and m denotes the number of states of the element X_j.

The posterior probability algorithm formula in Eq. (15) is as follows:

$$P(T = 0|X_j = 0) = \frac{P(T=0, X_j=0)}{P(X_j=0)} \tag{17}$$

Where,

$$P(T = 0|X_j = 0) = \sum_{X_0,X_1,\ldots,X_i} P(X_0 = x_0, X_1 = x_1, \ldots, X_i = x_i, T = 0, X_j = 0) \quad (i \neq j) \tag{18}$$

$$P(X_j = 0) = \sum_{X_0,X_1,\ldots,X_i} P(X_0 = x_0, X_1 = x_1, \ldots, X_i = x_i, X_j = 0) \quad (i \neq j) \tag{19}$$

The posterior probability *P (T = 0 |* X_j *= 0)* and the posterior probability *P (T = 0 |* X_j *= 1)* are given by Eq. (14) (17) (18) (19) .The two posteriori probabilities are brought into Eq. (16) to compute the importance *I (j)* of the component X_j.

4 CASE STUDIES: This chapter mainly uses the modeling method proposed in this paper to apply some typical systems to verify the effectiveness of the method. A typical natural gas pipeline compressor zone system is analyzed for the system reliability.

4.1 System introduction: A typical compressor area configuration diagram shown in Figure 3.

It can be seen from the figure, the natural gas from the compressor inlet tube input, respectively, through the three compressor lines, after the compressor pressurized, concentrated by the compressor outlet into the air cooler area through the air cooler cooling, and then to the downstream Piping. For each compressor line, there are inlet and outlet valves connected to the inlet and outlet piping, respectively, for the control of the function of the compressor line. The reliability diagram of the system is shown in Figure 4.

The compressor zone has three compressor lines which work together to achieve natural gas pressurization. The inlet and outlet valves in each compressor line are normally open and are closed when the pipe fails.

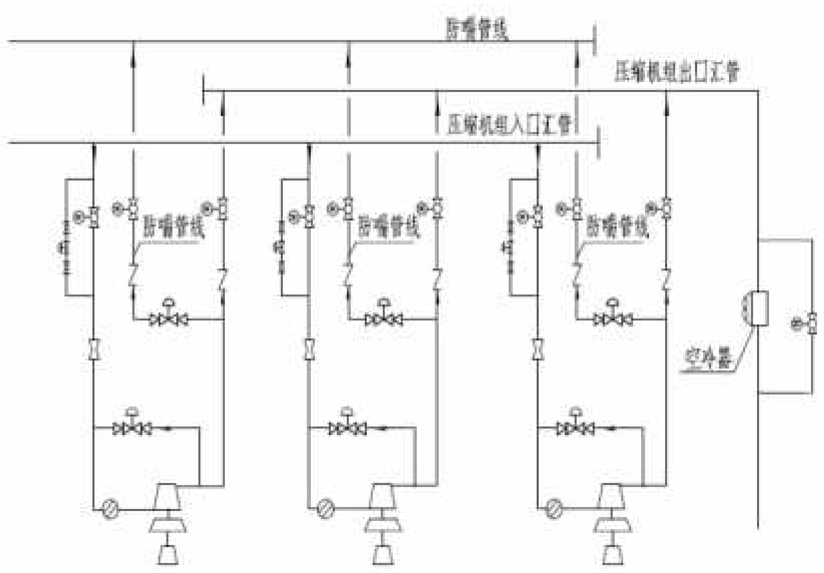

Figure 3: Typical compressor unit configuration diagram

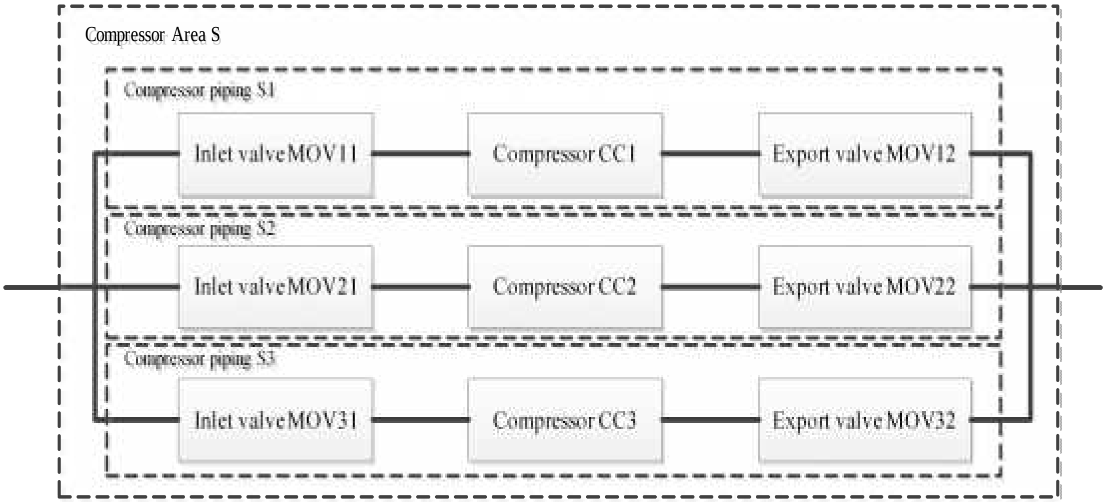

Figure 4: Typical Compressor Block Multi-State Reliability Block Diagram

4.2 Component state analysis: the compressor has two conditions by analysis, one is 3.75 million cubic meters per hour of gas transmission, and the other is 3 million cubic meters per hour, assuming its state as follows:

State 1: failure, the performance value is 0, the state probability is 0.0009.
State 2: condition 2, the performance value is 300, the state probability is 0.275.
State 3: condition 1, the performance value is 375, the state probability is 0.7241.

The universal generating function is:

$$U_{CC}(z) = 0.0009\,z^{0} + 0.275z^{300} + 0.7241z^{375} \tag{20}$$

The inlet and outlet valves have a variety of failure modes, corresponding to multiple states of the component, matching the multi-state system described in this article, after further failure mode analysis and we found that it has some failure states including leak, jam, cannot be turned off and cannot open. The different failure states have different effects on the system. From the failure of the point of view, these failure status and working conditions can be reduced to two states, so for the inlet valve and outlet valve, its state as follows:

State 1: leak, blocked, cannot be opened. The corresponding performance value is 0.
State 2: normal operation, the corresponding performance value is 1.

Assuming that the failure rate of the inlet and outlet valves is the same and the exponential distribution is obeyed, the probability of the two states is: state 1: $1-e^{-a_1 t}$; state 2: $e^{-a_1 t}$. Thus the general function of the inlet and outlet valves is:

$$u_{MOV}\left(z\right)=\left(1-e^{-a_1 t}\right)z^1+e^{-a_1 t}z^2 \tag{21}$$

When $a_1=1.9\times10^{-7}h^{-1}$ and t=87600 h, $u_{MOV}\left(z\right)=0.016506z^1+0.983494z^2$.

4.3 Reliability modeling and calculation of gas pipeline compressor area system based on Bayesian Network: Compressor line S3 circuit as a cold backup, the system dynamic operation obeys rules of m / N (w): G system. Each component life and maintenance time is subject to exponential distribution. According to the method described above to determine the failure mode of each unit and the conditional probability table shown as Table 5.

Table 2: Compressor area failure mode and data sheet

System/Component Name	Failure mode Name	Failure Rate h^{-1}	Maintenance Rate h^{-1}
Compressor area	Compressor area failure	/	/
Compressor piping	Compressor piping failure	/	/
Inlet valve	Inlet valve failure	3.64*10e-5	0.303
Export valve	Export valve failure	3.64*10e-5	0.303
Compressor	Compressor failure	2.47*10e-4	0.0641

The corresponding Bayesian Network is shown in Figure 5, set run time t = 8760 h:

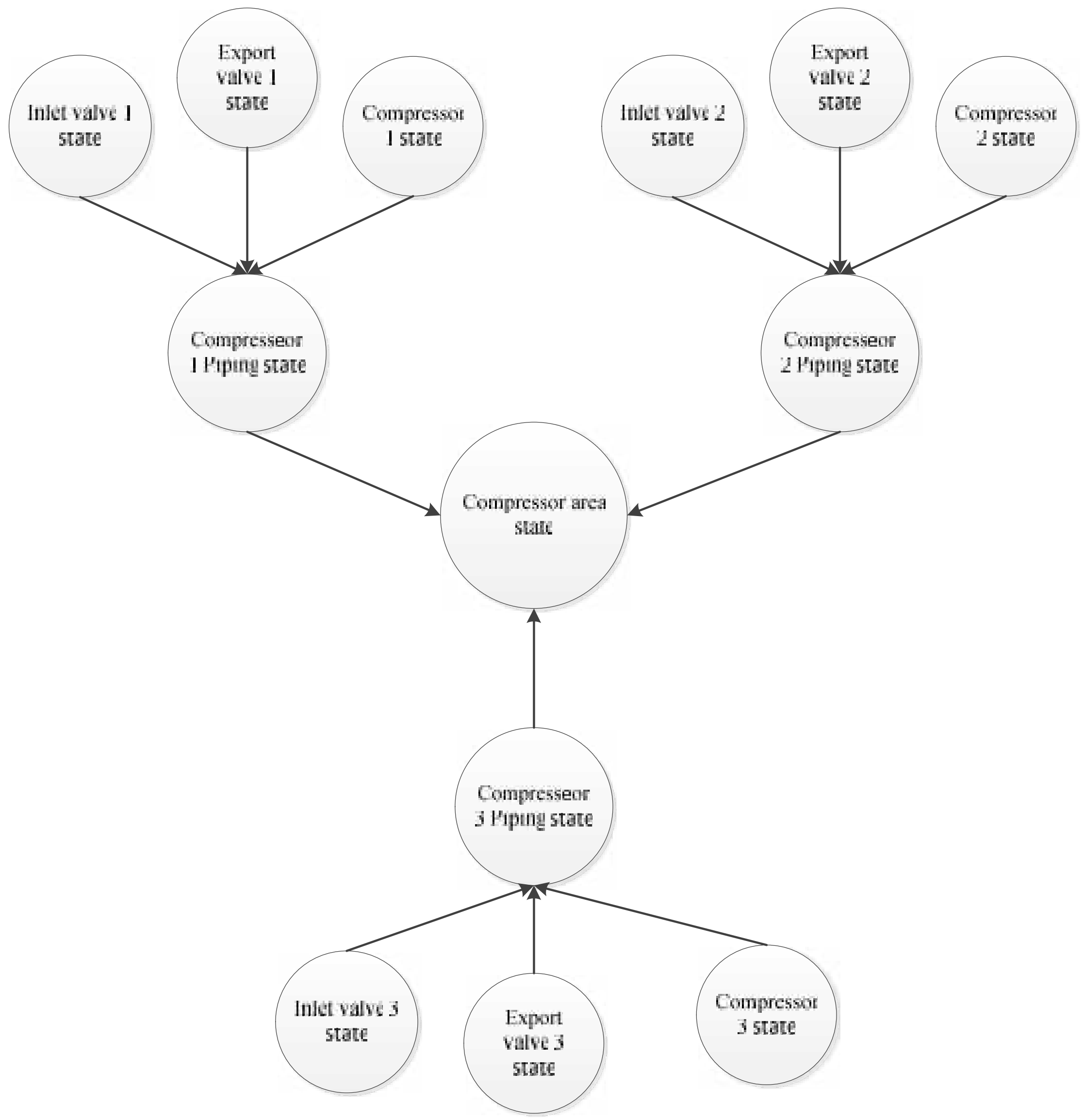

Figure 5: Bayesian network model of gas pipeline compressor area system

By using the forward predictive reasoning, the availability of compressor area system is 0.99551554. Based on the established Bayesian network model and the importance calculation method introduced in Section 3.3, the influence of the change of the bottom events on the total loss of function of the marine steam turbine generator system was obtained.

5 Conclusions: In this paper, Bayesian network and universal generating function is used to construct reliability evaluation model, and Bayesian network inference is used to get the probability of fault occurrence and the importance of each component in the system. It solves the polymorphism problem of probabilistic reliability evaluation of gas pipeline compressor area system. It is proved that this method is practical and feasible through the example of marine steam turbine generator system reliability evaluation. It can provide technical support for improving the system reliability of gas pipeline compressor area system.

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